

# PSYCHOMETRIC PRACTICES AT NBCE

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# DISCLAIMER

- ▶ This presentation is intended to introduce basic psychometric practices, which may or may not be implemented by NBCE.
- ▶ This presentation uses synthetic data. The statistics and graphics presented do not correspond to any of the NBCE tests.
- ▶ The purpose of this presentation is educational.

# CONTENT

- ▶ Classical Test Theory (CTT)
- ▶ Item Response Theory
  - ▶ 1 PL
  - ▶ 2 PL
  - ▶ 3 PL

# CLASSICAL TEST THEORY

$$X=T+E$$

- ▶ Where  $X$  is observed score
- ▶  $T$  is true score
- ▶  $E$  is error (systematic and random)
- ▶ The random error is assumed to average at 0

# CLASSICAL TEST THEORY

- ▶ Example:
- ▶ Sarah:  $X=7-2=5$
- ▶ John:  $X=5+3=8$
- ▶ Jane:  $X=8+0=8$



The errors are additive to the true score

# ITEM RESPONSE THEORY

- ▶ For binary item score with fixed ability:

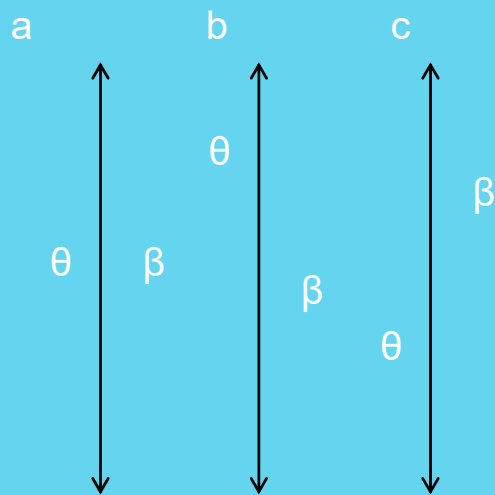
$$f_i(u_i|\theta) = P_i(\theta)^{u_i} Q_i(\theta)^{1-u_i}$$

$$f_i(u_i|\theta) = P_i(\theta) \text{ if } u_i = 1$$

$$f_i(u_i|\theta) = Q_i(\theta) \text{ if } u_i = 0$$

- ▶ ICC connects the means of these conditional distributions

# ITEM RESPONSE THEORY



- When the amounts are the same,  $P(Y_i = 1|\theta, \beta) = 0.5$
- When  $\theta > \beta$ ,  $P(Y_i = 1|\theta, \beta) > 0.5$
- When  $\theta < \beta$ ,  $P(Y_i = 1|\theta, \beta) < 0.5$

# ITEM RESPONSE THEORY (1PL)

- ▶ One-Parameter Logistic

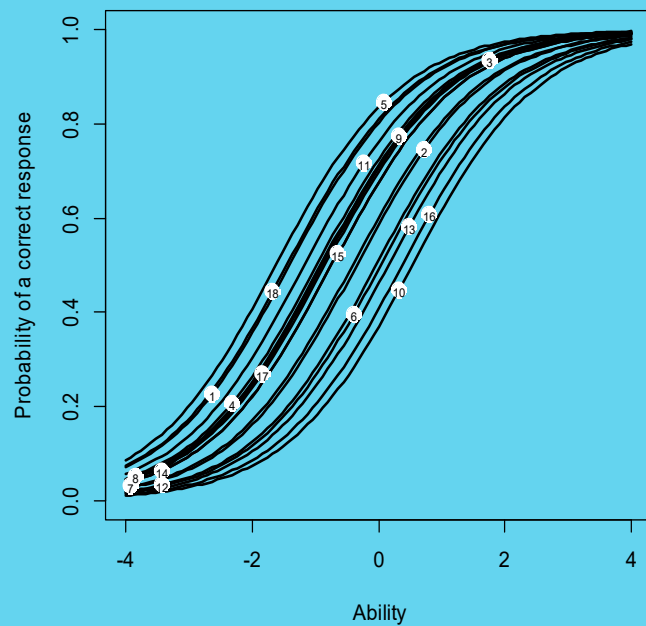
$$P_i(\theta) = \frac{e^{D(\theta-b_i)}}{1 + e^{D(\theta-b_i)}}$$

- ▶  $D$  is a scaling factor, set to  $D=1.7$ , then values of  $P(\theta)$  for 2-parameter Normal Ogive and 2PL differ absolutely by less than 0.01

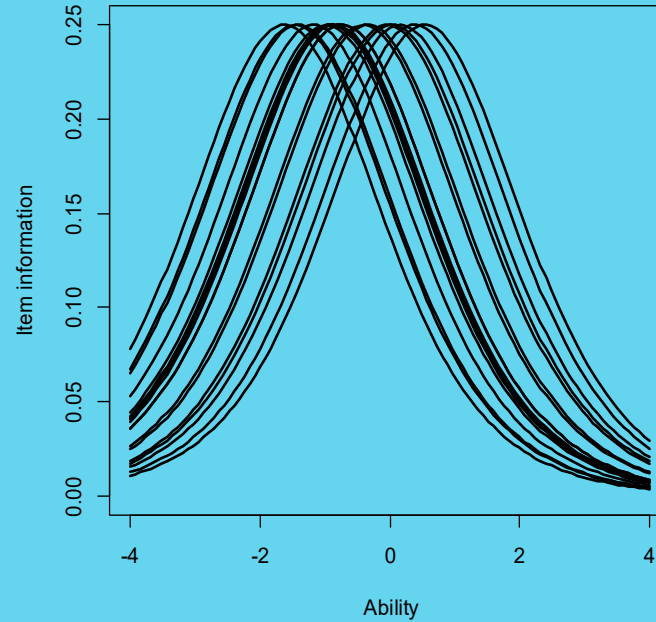


# ITEM RESPONSE THEORY (1PL)

Item response function



Item information function



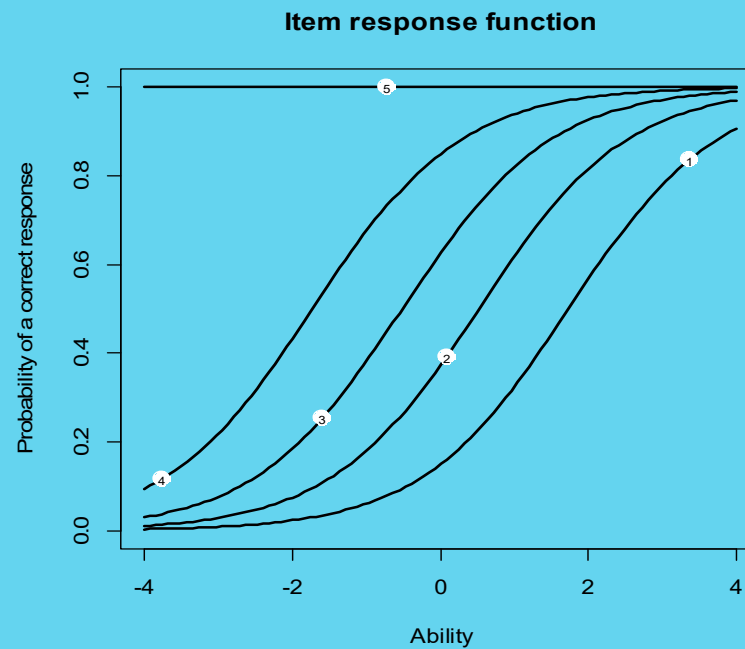
# ITEM RESPONSE THEORY

<i>Item</i>	<i>a</i>	<i>b</i>	<i>c</i>	<i>SE(a)</i>	<i>SE(b)</i>	<i>SE(c)</i>
1	1	-1.40916	0	NA	0.12678	0
2	1	-0.3503	0	NA	0.112833	0
3	1	-0.90295	0	NA	0.117894	0
4	1	-0.97388	0	NA	0.118879	0
5	1	-1.62649	0	NA	0.132039	0
6	1	0.025602	0	NA	0.111876	0
7	1	-0.41393	0	NA	0.11319	0
8	1	-0.87969	0	NA	0.117589	0
9	1	-0.91474	0	NA	0.118052	0
10	1	0.529859	0	NA	0.113682	0
11	1	-1.17102	0	NA	0.122048	0
12	1	-0.04723	0	NA	0.111909	0
13	1	0.150357	0	NA	0.11199	0
14	1	-0.74228	0	NA	0.115955	0
15	1	-0.7422	0	NA	0.115954	0
16	1	0.369904	0	NA	0.112718	0
17	1	-0.84488	0	NA	0.117147	0
18	1	-1.45123	0	NA	0.127726	0

# ITEM RESPONSE THEORY

## ► Rasch-Model-Based parameters

	Difficulty	SE
Item 1	1.73	1.31
Item 2	0.51	1.12
Item 3	-0.51	1.12
Item 4	-1.73	1.31
Item 5	-24.57	69881.01



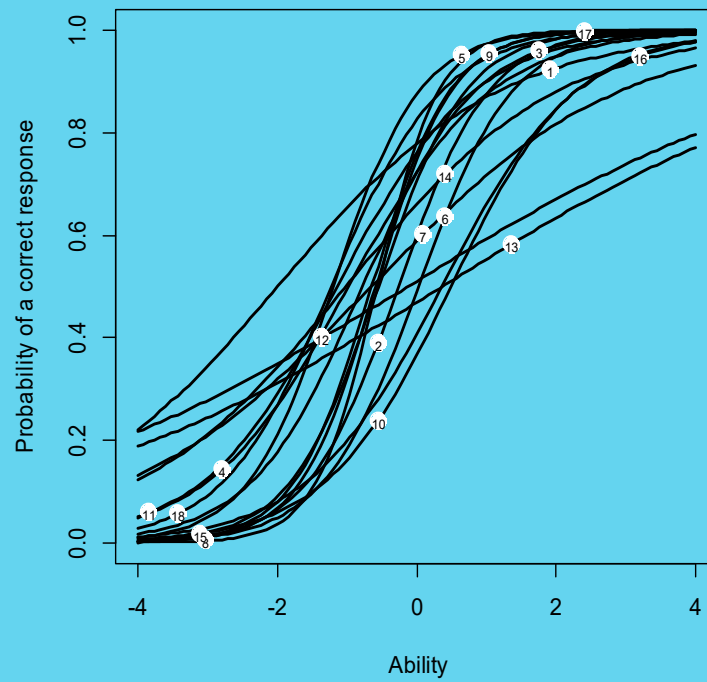
## ITEM RESPONSE THEORY (2PL)

- ▶ Two-Parameter Logistic

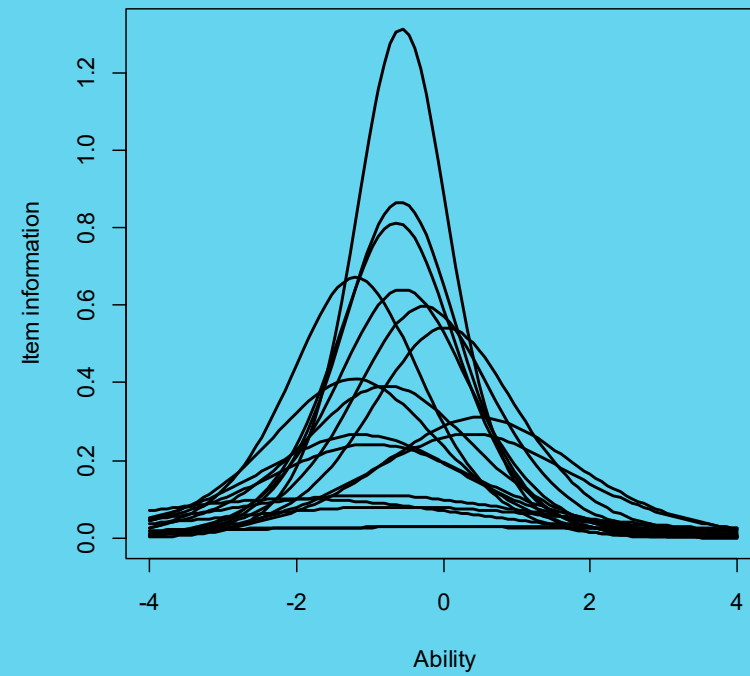
$$P_i(\theta) = \frac{e^{Da_i(\theta-b_i)}}{1 + e^{Da_i(\theta-b_i)}}$$

# ITEM RESPONSE THEORY (2PL)

Item response function



Item information function



# ITEM RESPONSE THEORY

<i>Item</i>	<i>a</i>	<i>b</i>	<i>c</i>	<i>SE(a)</i>	<i>SE(b)</i>	<i>SE(c)</i>
Item 1	0.633959	-1.99829	0	0.136118	0.409953	0
Item 2	1.544973	-0.26958	0	0.192543	0.087992	0
Item 3	1.251287	-0.775	0	0.169177	0.120588	0
Item 4	0.982795	-0.97929	0	0.148524	0.160425	0
Item 5	1.639096	-1.19563	0	0.226262	0.128849	0
Item 6	1.473177	0.017052	0	0.186232	0.087663	0
Item 7	0.56309	-0.65263	0	0.117878	0.214211	0
Item 8	2.291382	-0.56844	0	0.293392	0.079852	0
Item 9	1.802978	-0.64839	0	0.227331	0.091699	0
Item 10	1.114703	0.484064	0	0.156658	0.114729	0
Item 11	1.031583	-1.13696	0	0.155829	0.169759	0
Item 12	0.332253	-0.13094	0	0.108115	0.287453	0
Item 13	0.335735	0.363562	0	0.108386	0.303358	0
Item 14	0.661862	-1.01466	0	0.125967	0.227058	0
Item 15	1.600456	-0.55719	0	0.201144	0.094079	0
Item 16	1.034825	0.35369	0	0.148697	0.115295	0
Item 17	1.862107	-0.59097	0	0.232957	0.088195	0
Item 18	1.28049	-1.22317	0	0.183854	0.153317	0

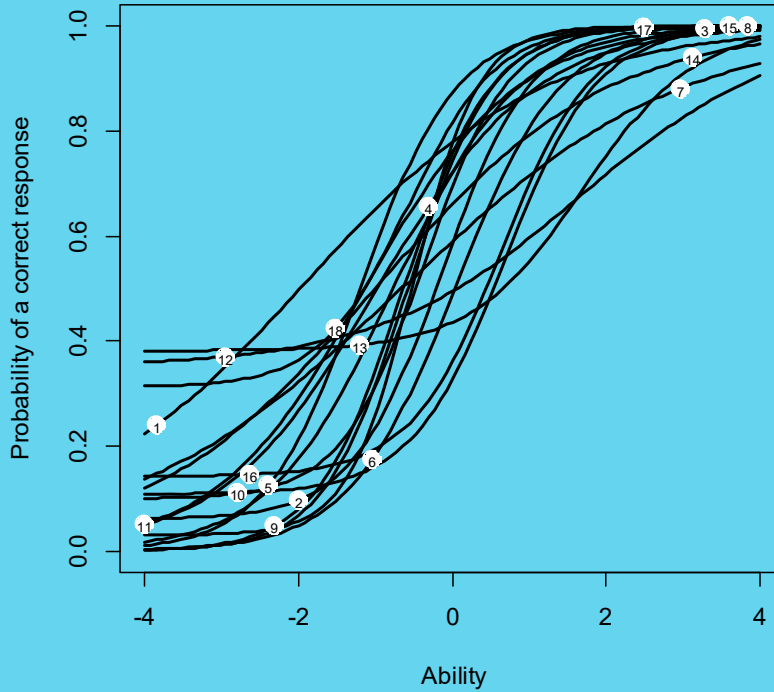
## ITEM RESPONSE THEORY (3PL)

- ▶ Three-Parameter Logistic

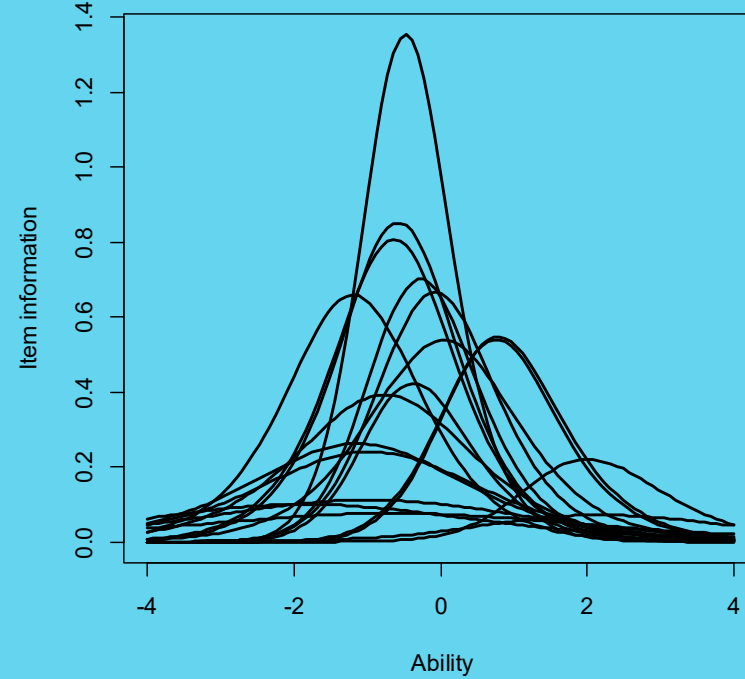
$$P_i(\theta) = c_i + (1 - c_i) \frac{e^{Da_i(\theta - b_i)}}{1 + e^{Da_i(\theta - b_i)}}$$

# ITEM RESPONSE THEORY (3PL)

Item response function



Item information function



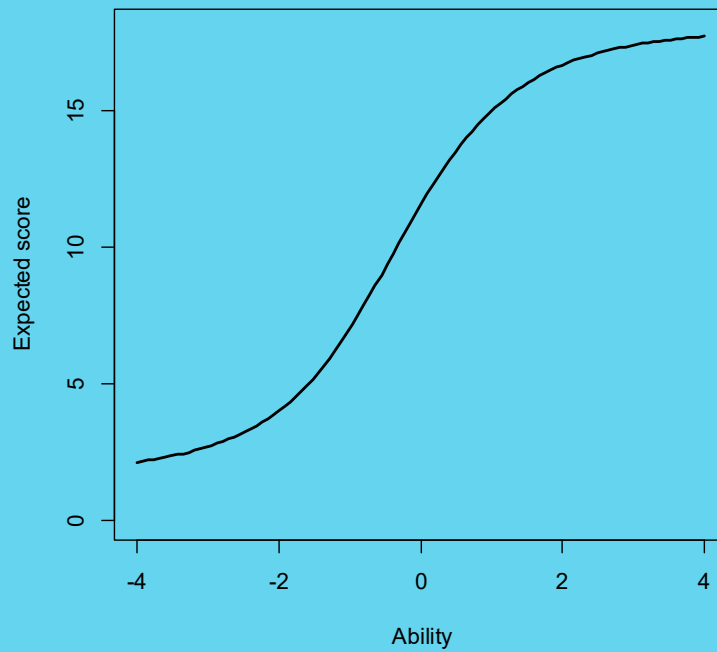


# ITEM RESPONSE THEORY

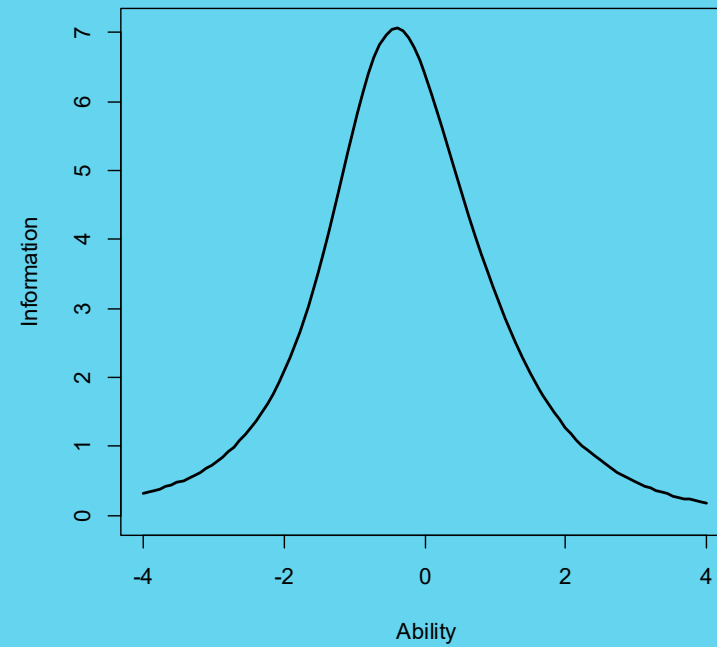
<i>Item</i>	<i>a</i>	<i>b</i>	<i>c</i>	<i>SE(a)</i>	<i>SE(b)</i>	<i>SE(c)</i>
Item 1	0.646375	-1.92136	0.019438	0.7685	7.971106	2.700359
Item 2	1.73303	-0.14405	0.060511	0.342198	0.16548	0.077618
Item 3	1.251851	-0.76673	8.44E-06	0.168204	0.121367	0.002308
Item 4	0.981489	-0.97489	4.37E-06	0.147121	0.161047	0.001606
Item 5	1.623707	-1.19519	2.55E-06	0.222949	0.130902	0.001296
Item 6	1.467363	0.026792	1.09E-06	0.185933	0.087758	0.000364
Item 7	0.55356	-0.65945	0.000118	0.117576	0.228725	0.017431
Item 8	2.396487	-0.50889	0.030139	0.487217	0.157415	0.090236
Item 9	1.794188	-0.63745	1.99E-06	0.226105	0.092784	0.000894
Item 10	1.640971	0.68005	0.108276	0.483988	0.149055	0.062981
Item 11	1.025076	-1.13699	6.41E-07	0.153912	0.171342	0.00035
Item 12	0.755559	1.661512	0.351234	0.793587	0.66428	0.195075
Item 13	1.3648	1.70745	0.381266	1.052467	0.409434	0.069082
Item 14	0.669582	-1.00169	2.10E-05	0.125995	0.224481	0.005063
Item 15	1.842625	-0.36759	0.099106	0.37052	0.188828	0.093887
Item 16	1.68479	0.632128	0.142288	0.419737	0.142096	0.053652
Item 17	1.844721	-0.58037	3.99E-05	0.233947	0.093924	0.017855
Item 18	1.757538	-0.58213	0.311964	0.470824	0.34171	0.146702

# ITEM RESPONSE THEORY (3PL)

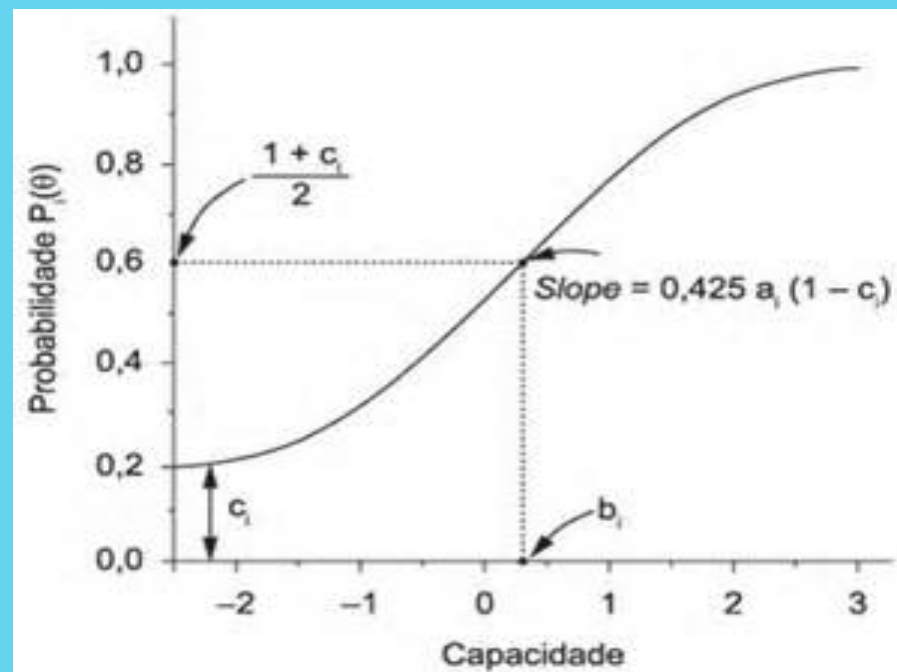
Test response function



Test information function



# ITEM RESPONSE THEORY



**Figura 1.** Curva característica da questão  $i$  para um modelo de três parâmetros. Fonte: Hambleton (1993).